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Public-Private Partnership in Regional Development as a Tool of Sustainable Management

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ABSTRACT

Public-private partnership is considered as a real instrument providing sustainable development of regional socio-ecological-economic systems. Many specific regional public-private partnership projects are analyzed in the literature. However, a long-term experience shows that an absence of the really interested control agents transforms all strategies of sustainable development into simple declarations without any practical importance. We use a new dynamic game theoretic model for description of consideration and coordination of interests of the agents responsible for regional sustainable management. Using both analytical investigation and computer simulation we found optimal values of such economic control parameters as shares of the total industrial investments in the gross product, shares of investments in public-private partnership projects, shares of inter-regional investments, and shares of participation of the agents in the profit from a public-private partnership project. We demonstrate a practical application of the model using official statistical data on the Southern Federal District of the Russian Federation, and the results may be used elsewhere.

KEYWORDS: public-private partnership; regional social-ecological-economic systems; Stackelberg games; sustainable management

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INTRODUCTION

According to the known definition the sustainable development is "a development that meets the needs of the present without compromising the ability of future generations to meet their own needs" [1]. This definition develops a concept of three pillars that determines three groups of obligatory conditions of viability that form a base for sustainable development: social, economic and ecological conditions. However, this generally recognized approach does not answer the key question about an agent of the sustainable development and its motivation. However, a long-term experience shows that an absence of the real control agent transforms all strategies of sustainable development into simple declarations without any practical importance. Therefore, an authors' concept of sustainable management postulates as necessary conditions not only viability but also consideration and coordination of interests of all active agents of sustainable development [2–4].

In the considered problem domain the main agents are society, power, and business. We treat the projects of public-private partnership as mechanisms of coordination of their interests. Public-Private Partnership (PPP) is a very wide-spread and prospective form of the enlistment of private firms to the solution of important tasks of development of the social and economic infrastructure on a regional level. In the PPP projects the society formulates problems and control their solution, the power organizes the whole process and provides the necessary conditions of its implementation, and the business partners realize the project tasks using their own and credit resources. Typical examples are environmental problems, construction of roads and social buildings, development of transport and communication networks.

European countries earlier than others started to develop the PPP sector [5,6], so still in 1981 in Great Britain was carried out a large-scale project of rebuilding the London's docks with participation of the private capital. In Russia, PPP mechanisms became widespread 20 years later and are mainly expressed in the formats of concession agreements. The leading positions in volume and scale of PPP projects are occupied by the following subjects: Moscow City, Nizhegorodskaya and Leningrad regions, implementing projects worth 150–200 billion rubles annually. The average volume of obligations under concession agreements in Russia is 1.6%–1.8% of the gross domestic product (GDP), while in Great Britain it is 6.6% of GDP, in Australia 6.9%, in Canada 8.1%, which clearly indicates the great potential of this mechanism for economic development.

A great complexity of the object increases essentially a worth of mathematical modeling of the problems of sustainable management and information technologies of their solution. The most adequate instrument of the consideration and coordination of interests of active agents is given by dynamic Stackelberg games [7,8].

In our previous publications we used Stackelberg games for the analysis of PPP projects and inter-regional interaction [9–11]. In those papers for description of a controlled dynamics of a regional social-ecological-economic system we used a known Solow model of economic growth [12] added by consideration of the environmental pollution and inter-regional interaction.

The contribution of this paper is the following: we conducted a system analysis of the problem of sustainable management with consideration of PPP projects and proposed the respective model hypotheses; on this base we built a mathematical model that is a complex dynamic conflict control model with a hierarchical structure and phase constraints; we received explicit form solutions of the game in some cases; we realized a numerical investigation of this model based on real statistical data on the Southern Federal District of the Russian Federation; we discussed the received results and possibilities of their practical application.

In the rest of the paper we present a literature review, we conduct a system analysis and formulate model hypotheses, we describe a mathematical model and its analytical investigation, we characterize a case study and numerical calculations, we discuss the results and conclude.

LITERATURE REVIEW

There is a lot of publications on sustainability science in the last half a century: see, for example [13–16]. Critical remarks and alternative concepts are presented in [17]. A mathematical formalization of the key viability conditions is given in a seminal monograph [18] and some consecutive papers [19–21].

Kinds of PPP, methods of their organization and financing, application domains are very different [22–26]. Specific Russian features of PPP projects are studied in [27,28]. The key role in PPP projects is played by the coordination of interests of their participants. Therefore, the most natural technique of modeling is provided by the theory of contracts [29–31]. There are many specific models of PPP [32–35], including Stackelberg games [36].

Paper [37] gives a systematic review of PPP projects for sustainable infrastructure development in Ghana. Paper [38] presents an empirical study of PPP infrastructure investment and sustainable economic development. Cui et al. [39] study the influence of passengers' perceived social responsibility efforts on their satisfaction in PPP urban rail transit projects. Liu et al. [40] provide a risk assessment of urban rail transit PPP project construction based on Bayesian network. De Matteis et al. [41] analyze PPP governance for accessible tourism in marine protected areas. Eshun et al. [42] give an evaluation of project risk dynamics in Sino-Africa public infrastructure delivery using an Interpretive Structural Modeling Approach. Guoyan et al. [43] use a quantile-on-quantile regression approach to evaluate the asymmetric relationship between PPP investment in energy and environmental degradation for sustainable development. Khan et al. [44] analyze the impact of technological innovation and PPP investment on sustainable environment in China. Kirikkaleli and Adebayo [45] answer whether renewable energy consumption and financial development matter for environmental sustainability.

However, these papers do not give a systematic representation and consideration of interests of the active agents of regional development.

An authors' theory of sustainable management is presented in [2–4]. Models of coordination of social and private interests in resource allocation (SPICE-models) are described in [46]. In [11] these models are applied to the PPP projects, and a detailed review is presented. A system analysis of the sustainable management on a regional level is made in [47,48]. Anopchenko et al. [9] describe an application of a modified Solow model of economic growth added by consideration of the environmental pollution and inter-regional interaction for solution of the problem of sustainable management of the Southern Federal District of the Russian Federation. Anopchenko et al. [10] consider a regional aspect of the PPP projects with consideration of the innovative economic development.

SYSTEM ANALYSIS AND MODEL HYPOTHESES

A conceptual framework of the hierarchical impact on a regional social-ecological-economic system with consideration of the requirements of sustainable development is shown in Figure 1.

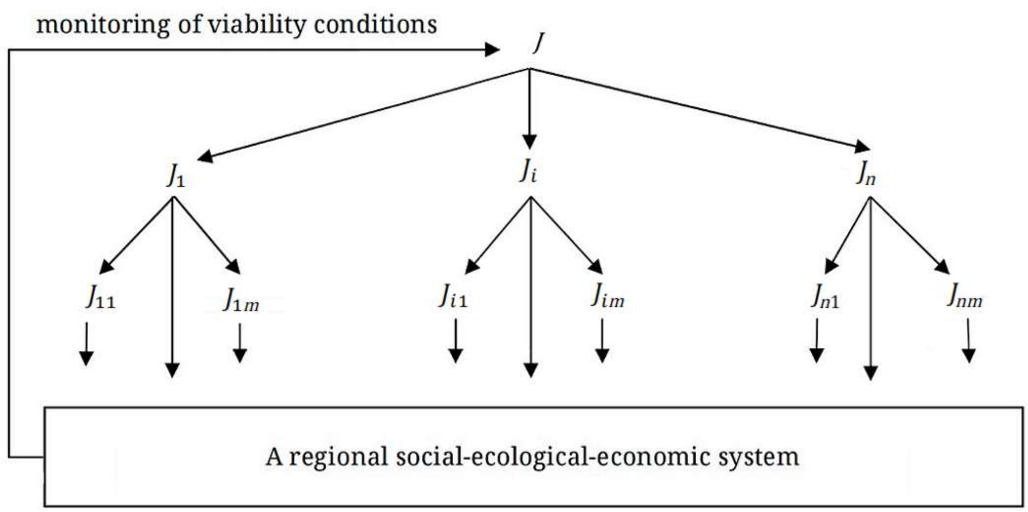

Figure 1. A conceptual framework of the hierarchical impact on a regional social-ecological-economic system with consideration of the requirements of sustainable development: J_0—principal (the federal state or its authorized coordinating body on a macro-regional level); J_i—supervisors (regional administrations); J_{ij}—agents (business partners on PPP projects).

The following hypotheses are accepted for mathematical formalization of this conceptual framework.

(1) The hierarchical impact on a regional social-ecological-economic system is made by a three-level control system. On its higher level the federal state or its authorized coordinating body on a macro-regional level (principal, he) is situated. On its middle level the regional administrations (supervisor, she) are situated. The lower level is presented by commercial firms (business partners) that participate in the PPP projects (agent, he).

(2) For description of dynamics of a regional social-ecological-economic system we use a known Solow model of economic growth [12] added by consideration of the hierarchical impact, environmental pollution, PPP projects and inter-regional financial interaction.

(3) An interaction between the higher and the middle control levels and between the middle and the lower levels is considered as a dynamic Stackelberg game of the type leader—n followers in open-loop strategies. Solutions of these games formalize a coordination of interests of the active participants of regional interaction as a necessary condition of sustainable development.

(4) Ecological conditions of viability mean that the total emissions of pollutants in the air and in the water do not exceed given threshold values (maximal permissible emissions). Economic conditions mean that the total gross product of the supervisors should be not less than a given threshold value. Social conditions are considered twofold. First, the principal's interest (reflected by his payoff functional) consists in the maximization of total consumption of the supervisors (or population of the respective regions). Second, the total payoff of the supervisors should be not less than a given threshold value. These conditions form the main principal's task that is formalized by phase constraints in his control problem.

(5) To provide the sustainable development the principal operates the following control variables for each supervisor: a share of the total industrial investments in the gross product; a share of investments in PPP projects; shares of investments in the economy of other regions; assignments in the environmental protection. Thus, the mentioned values are regulated by the federal state.

(6) A supervisor and her agents interact for the implementation of regional PPP projects. An agent distributes his resource between the implementation of a PPP project and other private activities. Respectively, his payoff is a sum of payoffs from the utility of a social good produced by the PPP projects and the income from private activities. A supervisor's interest consists in the maximization of the social good, and she determines the agents' shares in this good and a number of resources allocated for them.

(7) A supervisor's capital is divided on a general part and a part produced by the PPP projects. A dynamics of the first part is regulated by the principal in his game with supervisors, and a dynamics of the second part is regulated by the supervisors in their game with agents.

Mathematical Model

The mathematical model based on the enumerated hypotheses has the following form:

$$J_0 = \sum_{t=1}^{T} \delta^t \sum_{i=1}^{n} C_i(t) \to max \tag{1}$$

This expression equation (1) means that the principal's objective is expressed by maximization of the discounted sum of total annual regional consumption values.

$$0 \le s_i(t) \le 1 \tag{2}$$

The first group of principal's control variables are shares of industrial investments in the gross regional product (GDP) (equation (2)).

$$k_{ij}(t) \geq 0,\ \sum_{j=1}^{m_i} k_{ij} = 1 \tag{3}$$

In equation (3) the second group of principal's control variables are shares of GDP of the i-th supervisor invested to the economy of the j-th supervisor. Naturally, the sum of shares is equal to 1 (100%).

$$v_i^a(t) \geq 0,\ v_i^w(t) \geq 0 \tag{4}$$

Also, the principal determines the shares of GDP of the i-th supervisor assigned to the clearing of environmental pollution of the air and water respectively (equation (4)).

$$0 \leq B_i(t) \leq 1 \tag{5}$$

At last, the principal determines the shares of GDP of the i-th supervisor assigned to her PPP projects (equation (5)).

$$J_i = \sum_{t=1}^{T} \delta^t \sum_{j=1}^{m_i} a_{ij}[1 - s_{ij}(t)]g_{ij}(u_{ij}(t)) \to max \tag{6}$$

This expression (equation (6)) means that a supervisor's objective is expressed by maximization of the discounted sum of total revenues from PPP projects implemented by her business partners.

$$r_{ij}(t) \geq 0,\ \sum_{j=1}^{m_i} r_{ij}(t) = R_i(t) \tag{7}$$

Any regional administration allocates the respective budget $R_i(t)$ completely for the implementation of PPP projects by her business partners (equation (7)).

$$1 \geq s_{ij}(t) \geq 0 \tag{8}$$

Also, the regional administration reports to her business partners the shares of their participation in the income from PPP projects implementation (equation (8)).

$$J_{ij} = \sum_{t=1}^{T} \delta^t [p_{ij}(r_{ij}(t) - u_{ij}(t)) + s_{ij}(t)g_{ij}(u_{ij}(t))] \to max \tag{9}$$

According to the SPICE-methodology [46], the objective of any agent is expressed by maximization of the discounted sum of two terms: the mentioned above share of participation in the income from PPP projects and a revenue from the agent's private activity (equation (9)).

$$0 \leq u_{ij}(t) \leq r_{ij}(t) \tag{10}$$

An agent's control variable is a share of his resources assigned for the implementation of PPP projects (the rest goes to his private activity) (equation (10)).

$$Y_i(t) = A_i(t)[K_i(t)]^{\alpha_i}[E_i(t)L_i(t)]^{1-\alpha_i} \quad (11)$$

The i-th supervisor's GRP is calculated by a modified Cobb-Douglas function (equation (11)). Namely, the GRP depends on capital $K_i(t)$ and labor resources $L_i(t)$ with consideration of the labor efficiency $E_i(t)$ and scientific and technological progress $A_i(t)$. The parameter α_i characterizes the elasticity of production;

$$I_i(t) = s_i(t)Y_i(t) \quad (12)$$

$$R_i(t) = B_i(t)Y_i(t) \quad (13)$$

$$C_i(t) = [1 - s_i(t) - B_i(t)]Y_i(t) \quad (14)$$

These three equations describe an allocation of the GRP between investments (equation (12)), PPP implementation (equation (13)), and consumption (equation (14));

$$E_i(t+1) = (1+\eta_i)E_i(t),\ E_i(0) = E_{i0} \quad (15)$$

The equation (15) is a balance equation of the efficiency of labor resources with a coefficient η_i and given initial conditions;

$$L_i(t+1) = (1 + b_i - d_i)L_i(t),\ L_i(0) = L_{i0} \quad (16)$$

The equation (16) is a balance equation of the labor resources with coefficients of birth and mortality and given initial conditions;

$$K_i(t+1) = K_i^G(t+1) + \sum_{j=1}^{n} K_j^P(t+1) + \sum_{j=1}^{n} k_{ji}(t)I_j(t) \quad (17)$$

The i-th supervisor's capital in equation (17) is formed by her industrial capital, a capital produced by her PPP projects, and investments from other supervisors;

$$K_i^G(t+1) = (1-\mu_i)K_i^G(t),\ K_i^G(0) = K_{i0}^G \quad (18)$$

$$K_i^P(t+1) = (1-\mu_i)\sum_{j=1}^{m_i} g_{ij}^*(u_{ij}^*(t)),\ K_i^P(0) = K_{i0}^P \quad (19)$$

Equations (18) and (19) describe a balance of the mentioned above variables K_i^G and K_i^P with consideration of the depreciation and given initial conditions;

$$P_i^a(t) = [1 - c_i^a v_i^a(t)I_i(t)][B_{K_i}^a K_i(t) + B_{L_i}^a L_i(t)] \quad (20)$$

$$P_i^w(t) = [1 - c_i^w v_i^w(t)I_i(t)][B_{K_i}^w K_i(t) + B_{L_i}^w L_i(t)] \quad (21)$$

In these two equations we calculate an environmental pollution of the air (20) and water (21) respectively. The sources of pollution are economic activity (reflected by K_i) and population (L_i). We also consider an environmental clearing with the respective investments;

$$\sum_{t=1}^{T}\sum_{i=1}^{n} Y_i(t) \geq Y* \quad (22)$$

$$\sum_{t=1}^{T}\sum_{i=1}^{n} P_i^a(t) \leq P_a^* \tag{23}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{n} P_i^w(t) \leq P_w^* \tag{24}$$

$$\sum_{i=1}^{n} J_i \geq J* \tag{25}$$

The last four inequalities describe the conditions of sustainable development in economic (equation (22)), ecological (equations (23), (24)), and social (equation (25)) terms;

$$u_i(t) = (u_{i1}(t), \dots, u_{im_i}(t));\ t = 1, \dots, T;\ j = 1, \dots, m_i;\ i = 1, \dots, n \tag{26}$$

Here equations (1)–(5) are the principal's control problems subject to viability conditions (equations (22)–(25)); equations (6)–(8) are the i-th supervisor control problems; equation (9), equation (10) are the ij-th agent control problems; equations (11)–(17) are equations of dynamics of a regional social-ecological-economic system with additional assumptions.

The model denotations:

J_0, J_i, J_{ij} —payoff functionals of the principal, supervisors and agents respectively;

the principal's control variables relative to the i-th supervisor; s_i—a share of the total industrial investments in the gross product; B_i—a share of investments in PPP projects; k_{ij}—a share of investments of the i-th supervisor in the economy of the j-th supervisor (including her own when $i = j$); v_i^a, v_i^w—assignments in the environmental protection for air and water respectively;

supervisors' control variables relative to agents; r_{ij}—a resource (budget) allocated for an agent to the implementation of his PPP project; s_{ij}—a share of participation of the agent in the profit from the PPP project; agents' control variables; u_{ij}—a share of his resource allocated for the implementation of his PPP project;

i-th supervisor's state variables; Y_i—a gross product; I_i—total industrial investments; R_i—a budget of implementation of the PPP projects; C_i—a total consumption of the regional population; L_i—labor resources; E_i—an efficiency of their use; K_i—a total capital; K_i^P—a capital in PPP projects; K_i^G—a capital in other activities; P_i^a, P_i^w—emissions of pollution in the air and water respectively;

model functions: A_i—a function of influence of the innovative activity to the production of a gross product; g_i—a social good production function in a PPP project; p_{ij}—a function of income from other activities;

model parameters of the i-th supervisor: α_i—a parameter of the Cobb-Douglas production function; η_i—a parameter of growth of efficiency of labor resources; μ_i—a capital depreciation coefficient; a_i—a coefficient of efficiency in PPP projects; b_i, d_i —coefficients of reproduction and depreciation of the labor resources; c_i^a, c_i^w—coefficients of efficiency of the environmental protection; $B_{K_i}^a, B_{K_i}^w$ —unit emissions from industrial activities in the air and water respectively; $B_{L_i}^a, B_{L_i}^w$—unit emissions from households in the air and water respectively; $E_{i0}, L_{i0}, K_{i0}^G, K_{i0}^P$—given initial values of the respective variables;

general model parameters: T—a length of the considered period (in years); n—a total number of supervisors; m_i—a number of agents of the i-th supervisor; $\delta \in (0,1)$—a discount factor; $Y*, P_a^*, P_w^*, J*$—threshold values for viability.

The model consisting equations (1)–(25) is a complex dynamic control problem with phase constraints. It is analyzed as follows.

(1) Each supervisor $i = 1, \ldots, n$ solves a Stackelberg game including equations (6–10) in which the best response of the agents on the supervisor's strategy is a Nash equilibrium in the game of agents in normal form. It is supposed that $R_i(t) = \bar{R}_i, t = 1, \ldots, T, i = 1, \ldots, n$. It is explained by the fact that in their planning for the period $t = 1, \ldots, T$ the supervisors do not know their future budgets and base on a given initial value $\bar{R}_i$. A Stackelberg equilibrium in open-loop strategies is found:

$$\{r_{ij}^*(t), s_{ij}^*(t), u_{ij}^*(t)\}_{j=1}^{m_i}{}_{t=1}^{T}, i = 1, \ldots, n \tag{27}$$

The respective equilibrium values of the social good $g_i^*(u_i^*(t))$ are calculated and substituted in (19).

(2) The principal solves his problem described by equations (1)–(5), (11)–(21) with phase constraints described by equations (22)–(25). The best response of the supervisors on strategies described by equations (2)–(5) are equilibrium strategies in equation (27) found on the previous step with consideration of the agents' equilibrium strategies. Now values $R_i(t)$ are re-calculated by the equation (13) with consideration of all values of control variables and equations of dynamics.

Here the principal chooses optimal values of his control variables: shares of industrial investments in the GDP, shares of GDP of the i-th supervisor invested to the economics of the j-th supervisor, shares of GDP of the i-th supervisor assigned to the cleaning of environmental pollution of the air and water respectively, and shares of GDP of the i-th supervisor assigned to her PPP projects.

Analytical Investigation of the Model

In the model described by equations (1)–(25) we search for a Stackelberg equilibrium described by equation (19). Let us start from the problem described by equations (9), (10) on the lower control level. Notice that equations of dynamics (11)–(21) do not depend on the agents' actions, and their interests do not include the viability conditions described by equations (22)–(25). It means that the initial dynamic problem may be reduced to T static problems.

$$p_{ij}(r_{ij}(t) - u_{ij}(t)) + s_{ij}(t)g_{ij}(u_{ij}(t)) \to max \tag{28}$$

under constraint described by equation (10). A solution of this problem $u_{ij}^{max}(t)$ is the best response of an agent to his supervisor's strategies $s_{ij}(t)$ and $r_{ij}(t)$. Given the optimal reaction $u_{ij}^{max}(t)$ its substitution in equation (6) gives optimal strategies $s_{ij}(t)$, $r_{ij}(t)$ of the supervisor in her problem described by equations (6)–(8). In turn, this problem is reduced to T problems in the form:

$$\sum_{j=1}^{m_i} a_{ij}[1 - s_{ij}(t)]g_{ij}(u_{ij}^{max}(t)) \to max \quad (29)$$

$$r_{ij}(t) \geq 0, \sum_{j=1}^{m_i} r_{ij}(t) = \bar{R}_i \quad (30)$$

To find the values of $s_{ij}(t)$ each problem described by equation (29) may be decomposed on m_i subproblems in the form:

$$[1 - s_{ij}(t)]g_{ij}(u_{ij}^{max}(t)) \to max \quad (31)$$

because the values of $s_{ij}(t)$ exert influence only on the payoff of the agent j but not on the payoffs and strategies of other agents.

Thus, both regional administrations and their business partners choose the strategies which increase their payoffs on each step.

In spite of an essential simplification of the problem described by equations (6)–(10) and its reduction to T problems in the form described by equations (8), (10), (28)–(31), a specific form of the Stackelberg equilibrium depends on the form of functions $p_{ij}(\cdot)$ and $g_{ij}(\cdot)$. They are production functions, and we will take them in a linear or power concave form without an essential loss in generality.

Consider all four possible combinations of the functional forms $p_{ij}(\cdot)$ and $g_{ij}(\cdot)$: (1) both $p_{ij}(\cdot)$ and $g_{ij}(\cdot)$ are linear; (2) $p_{ij}(\cdot)$ are power concave, $g_{ij}(\cdot)$ are linear; (3) $p_{ij}(\cdot)$ are linear, $g_{ij}(\cdot)$ are power concave; (4) both $p_{ij}(\cdot)$ and $g_{ij}(\cdot)$ are power concave.

Case 1. Both functions $p_{ij}(\cdot)$ and $g_{ij}(\cdot)$ are linear, i.e., $p_{ij}(x) = p_{ij} \cdot x$, $g_{ij}(x) = g_{ij} \cdot x$.

In this case the problem described by equations (28), (10) has the form:

$$p_{ij} \cdot (r_{ij}(t) - u_{ij}(t)) + s_{ij}(t)g_{ij} \cdot u_{ij}(t) \to max \quad (32)$$

$$0 \leq u_{ij}(t) \leq r_{ij}(t) \quad (33)$$

Find a derivative of the payoff function by the variable u_{ij}:

$$-p_{ij} + s_{ij}(t)g_{ij} \quad (34)$$

It does not depend on u_{ij}, and has a constant value. Its sign determines an optimal value of $u_{ij}^{max}(t)$:

$$u_{ij}^{max}(t) = \begin{cases} r_{ij}(t), & s_{ij}(t)g_{ij} > p_{ij}, \\ 0, & s_{ij}(t)g_{ij} < p_{ij}. \end{cases} \quad (35)$$

Notice that if $r_{ij}(t) > 0$ then the model is sensitive to the values of parameters g_{ij} and p_{ij} as well as to the values of functions $s_{ij}(t)$. For example, suppose that $g_{ij} = 5, p_{ij} = 1.97$. If $s_{ij}(t) = 0.4$ (a business partner has 40% of the income) it holds that $s_{ij}(t)g_{ij} > p_{ij}$, and it is profitable for the agent to assign all his resources to the implementation of a PPP project but if $s_{ij}(t) = 0.39$ then the implementation of the PPP project becomes absolutely unprofitable. It is explained by the fact that the function $u_{ij}^{max}(t)$ has a gap when $s_{ij}(t)g_{ij} = p_{ij}$.

The best response of an agent on his supervisor's strategy is found. The agent allocates all his resources or only for production of a social good (social interests), or only for other activities (his private interests). We call agents of the first type individualists, and agents of the second type collectivists. Let us introduce two sets: a set of collectivists $C_i(t) = \{j | u_{ij}^{max}(t) = r_{ij}(t)\}$ and a set of individualists $I_i(t) = \{j | u_{ij}^{max}(t) = 0\}$. We will use these sets in the cases 2–4 also.

Come to supervisor's problem. Substitute equation (35) in equation (29). We receive the problem:

$$\sum_{j \in C_i} a_{ij}[1 - s_{ij}(t)]g_{ij} \cdot r_{ij}(t) \to max \tag{36}$$

$$r_{ij}(t) \geq 0, \sum_{j=1}^{m_i} r_{ij}(t) = \bar{R}_i \tag{37}$$

$$1 \geq s_{ij}(t) \geq 0 \tag{38}$$

Find optimal strategies $s_{ij}(t)$ first. Individualists are absent in equation (36). Therefore, a value of $s_{ij}(t)$ is indifferent for individualists because they do not produce a social good, and any share from zero is equal to zero. Thus, take $s_{ij}(t) = 0$. As for collectivists, the function described by equation (36) decreases on $s_{ij}(t)$. Therefore, it must be as small as possible such that the agent remains a collectivist because only they are present in equation (36). In the sum of equation (36) a positive summand corresponds to each agent-collectivist. Thus, the more summands, the better for the supervisor. It is profitable for a supervisor to provide the condition $s_{ij}(t)g_{ij} > p_{ij}$ for as many agents as possible. For an agent be a collectivist it is necessary that $s_{ij}(t) > \frac{p_{ij}}{g_{ij}}$ that is possible only if $p_{ij} < g_{ij}$. To attain maximum in equation (36), it is also necessary that $s_{ij}^{max}(t) = \frac{p_{ij}}{g_{ij}} + \varepsilon$. Thus,

$$s_{ij}^{max}(t) = \begin{cases} \frac{p_{ij}}{g_{ij}} + \varepsilon, & g_{ij} > p_{ij}, \\ 0, & g_{ij} < p_{ij}. \end{cases} \tag{39}$$

Notice that if $p_{ij} > 0$ then the model is sensitive to the changes of parameter values g_{ij} and p_{ij}. For example, if $g_{ij} = 5, p_{ij} = 5.01$ then $g_{ij} < p_{ij}$, and the supervisor does not give a PPP project to the agent but if $p_{ij} = 4.99$ then $g_{ij} > p_{ij}$, and the supervisor gives to the agent the project and the total income from its implementation. It is explained by the fact that the function $u_{ij}^{max}(t)$ has a gap when $s_{ij}(t)g_{ij} = p_{ij}$.

Substitute the found expression in equation (36) and receive an optimization problem for $r_{ij}(t)$:

$$\sum_{j\in C_i} a_{ij}\left[1-\frac{p_{ij}}{g_{ij}}\right]g_{ij}\cdot r_{ij}(t)=\sum_{j\in C_i} a_{ij}(g_{ij}-p_{ij})\cdot r_{ij}(t)\to max \quad (40)$$

$$r_{ij}(t)\geq 0,\ \sum_{j=1}^{m_i} r_{ij}(t)=\bar{R_i} \quad (41)$$

The problem described by equations (40), (30) is a linear programming problem that is solved as follows. Denote by k an index of such agent that a value $a_{ij}(g_{ij}-p_{ij})$ is maximal: $k=\arg\max_{j\in C_i}\{a_{ij}(g_{ij}-p_{ij})\}$. Notice that k does not depend on time t. Then,

$$r_{ij}^{max}(t)=\begin{cases}\bar{R_i}, & j=k,\\ 0, & j\neq k.\end{cases} \quad (42)$$

Thus, if both functions of social and private income are linear then a supervisor gives the whole budget to an only agent that brings her the maximal payoff. Other agents do not produce a social good and do not take part in its distribution. Besides, strategies $s_{ij}^{max}(t)$ and $r_{ij}^{max}(t)$ do not depend on time. Therefore, agents' strategies $u_{ij}^{max}(t)$ also do not depend on time. Thus, neither supervisors nor agents change their strategies in time.

Notice that a regional administration's strategy $r_{ij}^{max}(t)$ is sensitive to the parameters a_{ij}, g_{ij} and p_{ij}. A small exceeding of the term $a_{ij}(g_{ij}-p_{ij})$ of one agent over the respective terms of other agents changes sharply an allocation of the supervisor's resources between the agents. Agents' payoff:

$$J_{ij}=\begin{cases}[p_{ij}+\varepsilon g_{ij}]\sum_{t=1}^{T}\delta^t R_i(t)\to max, & i=k, p_{ik}<g_{ik},\\ 0, & otherwise\end{cases} \quad (43)$$

Supervisor's payoff:

$$J_i=a_{ik}[g_{ik}-p_{ik}-\varepsilon g_{ik}]\sum_{t=1}^{T}\delta^t\cdot R_i(t) \quad (44)$$

Case 2. Functions $p_{ij}(\cdot)$ are power concave, $g_{ij}(\cdot)$ are linear, i.e., $p_{ij}(x)=p_{ij}\cdot x^\alpha$, $g_{ij}(x)=g_{ij}\cdot x$, $0<\alpha<1$.

In this case the problem described by equations (28), (10) has the form:

$$p_{ij}\cdot(r_{ij}(t)-u_{ij}(t))^\alpha+s_{ij}(t)g_{ij}\cdot u_{ij}(t)\to max \quad (45)$$

$$0\leq u_{ij}(t)\leq r_{ij}(t) \quad (46)$$

A first order condition has the form:

$$\frac{p_{ij}\alpha}{(r_{ij}(t)-u_{ij}(t))^{1-\alpha}}=s_{ij}(t)g_{ij} \quad (47)$$

It determines an optimal strategy $u_{ij}^{max}(t)$:

$$u_{ij}^{max}(t) = \begin{cases} r_{ij}(t) - \sqrt[1-\alpha]{\dfrac{p_{ij}\alpha}{s_{ij}(t)g_{ij}}}, & \dfrac{p_{ij}\alpha}{s_{ij}(t)g_{ij}} < r_{ij}^{1-\alpha}(t), \\ 0, & \dfrac{p_{ij}\alpha}{s_{ij}(t)g_{ij}} > r_{ij}^{1-\alpha}(t). \end{cases} \quad (48)$$

Notice that here the model is not sensitive to the model parameters g_{ij} and p_{ij} as well as to the functions $s_{ij}(t)$. It is explained by the fact that the function $u_{ij}^{max}(t)$ is continuous because in the condition of inflexion point $\frac{p_{ij}\alpha}{s_{ij}(t)g_{ij}} = r_{ij}^{1-\alpha}(t)$ an expression $r_{ij}(t) - \sqrt[1-\alpha]{\frac{p_{ij}\alpha}{s_{ij}(t)g_{ij}}}$ is equal to 0.

The best response of an agent on his supervisor's strategy is found. Notice that in this case collectivists are absent because an expression under the square root in equation (48) is positive. Introduce an auxiliary set: $C_i'(t) = \{j | 0 < u_{ij}^{max}(t) < r_{ij}(t)\}$ of agents that are neither individualists nor collectivists. Such agents use a part of their resources in social interests, and the rest in their private interests.

Come to supervisor's problem. Substitute equation (48) in equation (36). We receive the problem:

$$\sum_{j \in C_i'(t)} a_{ij}[1 - s_{ij}(t)]g_{ij} \cdot \left(r_{ij}(t) - \sqrt[1-\alpha]{\frac{p_{ij}\alpha}{s_{ij}(t)g_{ij}}} \right) \to max \quad (49)$$

$$r_{ij}(t) \geq 0,\ \sum_{j=1}^{m_i} r_{ij}(t) = \bar{R}_i;\ 1 \geq s_{ij}(t) \geq 0 \quad (50)$$

Find optimal strategies $s_{ij}(t)$ first. As in the previous case, individualists are absent in equation (49), and a value of $s_{ij}(t)$ is indifferent for them: we take $s_{ij}(t) = 0$. As for non-individualists, for them the problem described by equations (49), (30), (8) is decomposed on m_i problems in the form:

$$[1 - s_{ij}(t)] \cdot \left(r_{ij}(t) - \sqrt[1-\alpha]{\frac{p_{ij}\alpha}{s_{ij}(t)g_{ij}}} \right) \to max \quad (51)$$

$$1 \geq s_{ij}(t) \geq 0 \quad (52)$$

The first order condition has the form:

$$-r_{ij}(t) + \sqrt[1-\alpha]{\frac{p_{ij}\alpha}{s_{ij}(t)g_{ij}}} + [1 - s_{ij}(t)] \cdot \frac{1}{1-\alpha} \cdot \sqrt[1-\alpha]{\frac{p_{ij}\alpha}{s_{ij}^{2-\alpha}g_{ij}}} = 0 \quad (53)$$

For solution of equation (53) we can apply any appropriate numerical method or find an approximate solution with a required precision by enumeration. However, to solve the equation (53) it is necessary to know the value of $r_{ij}(t)$.

As for optimal values of $r_{ij}(t)$, we notice that the problem described by equations (49), (30) is a linear programming problem that is solved similarly to the previous case. Denote by k an index of the agent such that the value of $a_{ij}(1 - s_{ij})g_{ij}$ is maximal: $k = \arg\max_{j \in C_i}\{a_{ij}(1 - s_{ij})g_{ij}\}$. Then:

$$r_{ij}^{max}(t) = \begin{cases} \bar{R}_i, & j = k, \\ 0, & j \neq k. \end{cases} \quad (54)$$

Thus, the supervisor allocates all resources to the agent-collectivist with a maximal value $a_{ij}(1-s_{ij})g_{ij}$. The supervisor's strategy $r_{ij}^{max}(t)$ is sensitive to the parameters a_{ij}, g_{ij} and s_{ij}. A small exceeding of the term $a_{ij}(g_{ij}-p_{ij})$ of one agent over the respective terms of other agents changes sharply an allocation of the supervisor's resources between the agents.

The problem here is that for determination of k it is required to know s_{ij}, and to find them from equation (53) it is necessary to know r_{ij}.

Let us use the fact that a strategy $r_{ij}^{max}(t)$ takes only two feasible values. Then for each agent j we can find from equation (53) the respective supervisor's strategy $s_{ij}^{max}(t)$ assuming that $r_{ij}(t)=\bar{R}_i$, and choose from all agents the index $k=j$ if the value of $a_{ij}(1-s_{ij}^{max}(t))g_{ij}$ is maximal. Notice that here a strategy $s_{ij}^{max}(t)$ depends on time because it depends on a specific value of $r_{ij}(t)$.

Case 3. Functions $p_{ij}(\cdot)$ are linear, $g_{ij}(\cdot)$ are power concave, i.e., $p_{ij}(x)=p_{ij}\cdot x$, $g_{ij}(x)=g_{ij}\cdot x^{\alpha}$, $0<\alpha<1$.

In this case the problem described by equations (28), (10) has the form:

$$p_{ij}\cdot(r_{ij}(t)-u_{ij}(t))+s_{ij}(t)g_{ij}\cdot u_{ij}(t)^{\alpha}\to max \tag{55}$$

$$0\le u_{ij}(t)\le r_{ij}(t) \tag{56}$$

The first order condition has the form:

$$p_{ij}=\frac{\alpha s_{ij}(t)g_{ij}}{u_{ij}(t)^{1-\alpha}} \tag{57}$$

It determines an optimal value of $u_{ij}^{max}(t)$:

$$u_{ij}^{max}(t)=\begin{cases} r_{ij}(t), & \frac{\alpha s_{ij}(t)g_{ij}}{p_{ij}}>r_{ij}^{1-\alpha}(t),\\ \sqrt[1-\alpha]{\frac{\alpha s_{ij}(t)g_{ij}}{p_{ij}}}, & \frac{\alpha s_{ij}(t)g_{ij}}{p_{ij}}<r_{ij}^{1-\alpha}(t). \end{cases} \tag{58}$$

Notice that here the model is not sensitive to the model parameters g_{ij} and p_{ij} as well as to the functions $s_{ij}(t)$. It is explained by the fact that the function $u_{ij}^{max}(t)$ is continuous because in the condition of inflexion point $\frac{\alpha s_{ij}(t)g_{ij}}{p_{ij}}=r_{ij}^{1-\alpha}(t)$ an expression $\sqrt[1-\alpha]{\frac{\alpha s_{ij}(t)g_{ij}}{p_{ij}}}$ is equal to $r_{ij}(t)$.

The best response of an agent to his supervisor's strategy is found. Notice that in this case, as distinct from the Case 2, individualists are absent because an expression under the root in equation (58) is positive.

Come to supervisor's problem. Substitute equation (35) in equation (29). We receive the problem:

$$\sum_{j\in C_i(t)}a_{ij}[1-s_{ij}(t)]g_{ij}\cdot\left(\sqrt[1-\alpha]{r_{ij}(t)}\right)^{\alpha}+\sum_{j\in C_i'(t)}a_{ij}[1-s_{ij}(t)]g_{ij}\cdot\left(\sqrt[1-\alpha]{\frac{\alpha s_{ij}(t)g_{ij}}{p_{ij}}}\right)^{\alpha}\to max \tag{59}$$

$$r_{ij}(t)\ge 0,\ \sum_{j=1}^{m_i}r_{ij}(t)=\bar{R}_i \tag{60}$$

$$1\ge s_{ij}(t)\ge 0 \tag{61}$$

Find optimal strategies $s_{ij}(t)$ first. For collectivists $s_{ij}(t)$ should be as small as possible (as the respective summand in equation (57) decreases by $s_{ij}(t)$) but the agent must remain a collectivist, i.e., $s_{ij}(t) = \frac{r_{ij}^{1-\alpha}(t)p_{ij}}{\alpha g_{ij}} + \varepsilon$. It is possible if $r_{ij}^{1-\alpha}(t)p_{ij} < \alpha g_{ij}$. As for non-collectivists, the first order condition gives

$$-\left(\sqrt[1-\alpha]{\frac{\alpha s_{ij}(t)g_{ij}}{p_{ij}}}\right)^{\alpha} + [1 - s_{ij}(t)] \cdot \frac{\alpha}{1-\alpha}\left(\sqrt[1-\alpha]{\frac{\alpha g_{ij}}{p_{ij}}}\right)^{\alpha} s_{ij}^{\frac{\alpha}{1-\alpha}-1}(t) = 0 \quad (62)$$

A division by the respective non-zero value $\left(\sqrt[1-\alpha]{\frac{\alpha g_{ij}}{p_{ij}}}\right)^{\alpha}$ implies

$$\left(\sqrt[1-\alpha]{s_{ij}(t)}\right)^{\alpha}\left(-1 + \frac{\alpha}{1-\alpha}\left(\frac{1}{s_{ij}(t)} - 1\right)\right) = 0 \quad (63)$$

Thus, $s_{ij}(t) = 0$ or $s_{ij}(t) = \alpha$. The second order conditions show that $s_{ij}(t) = 0$ is a minimum point, and $s_{ij}(t) = \alpha$ is a maximum point. A substitution of the received values of $s_{ij}(t)$ in equation (58) gives

$$\begin{gathered}\sum_{j \in C_i(t)} a_{ij}\left(1 - \frac{r_{ij}^{1-\alpha}(t)p_{ij}}{\alpha g_{ij}} - \varepsilon\right) g_{ij} \cdot r_{ij}^{\frac{\alpha}{1-\alpha}-1} + \\ \sum_{j \in C_{i'}(t)} a_{ij}(1-\alpha)g_{ij} \cdot \left(\sqrt[1-\alpha]{\frac{\alpha s_{ij}(t)g_{ij}}{p_{ij}}}\right)^{\alpha} = \\ \sum_{j \in C_i(t)} \frac{a_{ij}}{\alpha}\left((1-\varepsilon)\alpha g_{ij} - r_{ij}^{1-\alpha}(t)p_{ij}\right) \cdot r_{ij}^{\frac{\alpha}{1-\alpha}-1} + \\ \sum_{j \in C_{i'}(t)} a_{ij}(1-\alpha)g_{ij} \cdot \left(\sqrt[1-\alpha]{\frac{\alpha s_{ij}(t)g_{ij}}{p_{ij}}}\right)^{\alpha} \to max\end{gathered} \quad (64)$$

It is seen that all resources should be distributed only among collectivists with consideration of the condition described by equation (30). A further analytical investigation is impossible.

Case 4. Both functions $p_{ij}(\cdot)$ and $g_{ij}(\cdot)$ are power concave, i.e., $p_{ij}(x) = p_{ij} \cdot x^{\alpha}$, $g_{ij}(x) = g_{ij} \cdot x^{\alpha}$, $0 < \alpha < 1$.

In this case the problem described by equations (28), (10) has the form:

$$p_{ij} \cdot (r_{ij}(t) - u_{ij}(t))^{\alpha} + s_{ij}(t)g_{ij} \cdot u_{ij}(t)^{\alpha} \to max \quad (65)$$

$$0 \le u_{ij}(t) \le r_{ij}(t) \quad (66)$$

The first order condition has the form:

$$\frac{p_{ij}}{(r_{ij}(t) - u_{ij}(t))^{1-\alpha}} = \frac{s_{ij}(t)g_{ij}}{u_{ij}(t)^{1-\alpha}} \quad (67)$$

It determines an optimal value of $u_{ij}^{max}(t)$:

$$u_{ij}^{max}(t) = \frac{\sqrt[1-\alpha]{s_{ij}(t)g_{ij}}}{\sqrt[1-\alpha]{p_{ij}} + \sqrt[1-\alpha]{s_{ij}(t)g_{ij}}} r_{ij}(t) \quad (68)$$

The best response of an agent to his supervisor's strategy is found. Notice that in this case there are neither individualists nor collectivists.

Come to supervisor's problem. Substitute equation (68) in equation (47). We receive the problem:

$$\sum_{j=1}^{m_i} a_{ij}[1-s_{ij}(t)]g_{ij} \cdot \left(\frac{\sqrt[1-\alpha]{s_{ij}(t)g_{ij}}}{\sqrt[1-\alpha]{p_{ij}} + \sqrt[1-\alpha]{s_{ij}(t)g_{ij}}}\right)^{\alpha} \cdot r_{ij}(t)^{\alpha} \to max \quad (69)$$

$$r_{ij}(t) \geq 0,\ \sum_{j=1}^{m_i} r_{ij}(t) = \bar{R}_i \quad (70)$$

$$1 \geq s_{ij}(t) \geq 0 \quad (71)$$

Thus, Cases 1 and 2 allow for a complete analytical investigation while Cases 3 and 4 do not. The problem is complicated by the fact that in the expressions for calculation $s_{ij}(t)$ and $r_{ij}(t)$ values of $s_{ij}(t)$ depend on $r_{ij}(t)$, and $r_{ij}(t)$ depend $s_{ij}(t)$. May be it is possible to solve the problem described by equations (29), (30) given $s_{ij}(t)$ or the problem described by equations (31), (8) given $r_{ij}(t)$. From an economic point of view that means a separate use of mechanisms of stimulation by resource allocation or by participation in the common income. We leave this idea for our future investigations.

Now come to the principal's problem. He solves the problem described by equation (1) with constraints described by equations (2)–(5) and equations of system dynamics (11)–(21) together with viability conditions. In the principal's payoff function:

$$J_0 = \sum_{t=1}^{T} \delta^t \sum_{i=1}^{n} C_i(t) \to max \quad (72)$$

We substitute equation (14):

$$J_0 = \sum_{t=1}^{T} \delta^t \sum_{i=1}^{n} C_i(t) = \sum_{t=1}^{T} \delta^t \sum_{i=1}^{n} [1 - s_i(t) - B_i(t)]Y_i(t) \quad (73)$$

In turn, substitute here equation (11):

$$J_0 = \sum_{t=1}^{T} \delta^t \sum_{i=1}^{n} [1 - s_i(t) - B_i(t)]A_i(t)[K_i(t)]^{\alpha_i}[E_i(t)L_i(t)]^{1-\alpha_i} \quad (74)$$

In the resulting expression we substitute equation (17):

$$J_0 = \sum_{t=1}^{T} \delta^t \sum_{i=1}^{n} [1 - s_i(t) - B_i(t)]A_i(t) \times$$
$$[K_i^G(t+1) + \sum_{j=1}^{n} K_j^P(t+1) + \sum_{j=1}^{n} k_{ji}(t)I_j(t)]^{\alpha_i}[E_i(t)L_i(t)]^{1-\alpha_i} \quad (75)$$

and now substitute equations (19) and (12):

$$J_0 = \sum_{t=1}^{T} \delta^t \sum_{i=1}^{n} [1 - s_i(t) - B_i(t)] A_i(t) \times \\ [K_i^G(t+1) + \sum_{j=1}^{n} K_j^P(t+1) + \sum_{j=1}^{n} k_{ji}(t) I_j(t)]^{\alpha_i} [E_i(t) L_i(t)]^{1-\alpha_i} \quad (76)$$

In the part $g_{ij}^*(u_{ij}^*(t))$ of the received expression there is a dependency on $R_i(t)$ or on $B_i(t)Y_i(t)$.Thus, we will investigate the model numerically.

Case Study and Numerical Calculations

Practical testing of the model was carried out on materials of cross-border interaction of the regions of the Southern Federal District (SFD) of the Russian Federation, which include the Rostov region (in the model the index $i = 1$ is assigned), the Volgograd region ($i = 2$), the Krasnodar region ($i = 3$), the Republic of Adygea ($i = 4$), the Astrakhan region ($i = 5$), the Republic of Kalmykia ($i = 6$), the Republic of Crimea ($i = 7$). For the study of cross-border interactions of regions within the SFD, it is necessary to identify the parameters vectors of the model for each region, and also to develop a computer program for calculating the main indicators, specifically Y_i, C_i, I_i, K_i, L_i, R_i, E_i for a time period of 5 years.

The model parameters were identified according to data from the Federal State Statistics Service of the Russian Federation for 2005, 2010, 2015–2017 years. These time intervals were determined based on the results of stable positive dynamics of macroeconomic growth and in order to exclude the extreme negative influence of global pre-crisis economic trends of 2008, the systematic consequences of the socio-political Russian crisis of 2013 and the global period of socio-epidemiological instability due to the 2019–2022 coronavirus pandemic.

The model parameters, in particular the coefficient $A_i(t)$, were identified according to statistical data from the Russian Federation [9–11].

1) Parameter α_i of the Cobb-Douglas production function (Table 1):

Table 1. Parameters α_i of the Cobb-Douglas function of the SFD regions.

Region	Parameter	Value
Rostov region	α_1	0.214360
Volgograd region	α_2	0.107626
Krasnodar region	α_3	0.144864
Republic of Adygea	α_4	0.236457
Astrakhan region	α_5	0.078457
Republic of Kalmykia	α_6	0.145812
Republic of Crimea	α_7	0.150498

2) For calculation of the difference between the reproduction rate and the rate of depreciation of labor resources ($b_i - d_i$) data of natural increment by region for selected periods were used, from which the arithmetic mean was formed (Table 2).

Table 2. Parameters of the natural growth function of the SFD regions.

Region	Parameter	Value
Rostov region	$b_1 - d_1$	−0.0005
Volgograd region	$b_2 - d_2$	−0.01775
Krasnodar region	$b_3 - d_3$	0.00575
Republic of Adygea	$b_4 - d_4$	−0.008
Astrakhan region	$b_5 - d_5$	0.00375
Republic of Kalmykia	$b_6 - d_6$	−0.00375
Republic of Crimea	$b_7 - d_7$	−0.007

3) For determination of the parameter of growth of the efficiency of labor resources η_i , labor productivity data $E_i(t)$ for the corresponding periods is used, which is determined by the ratio of the value of gross product $Y_i(t)$ and the size of the working population $L_i(t)$ (Table 3):

Table 3. Parameters η_i of the SFD regions.

Region	Parameter	Value
Rostov region	η_1	0.009251
Volgograd region	η_2	−0.017337
Krasnodar region	η_3	0.024048
Republic of Adygea	η_4	0.001221
Astrakhan region	η_5	0.027663
Republic of Kalmykia	η_6	0.002532
Republic of Crimea	η_7	0.017375

Note: Table includes some of the factors from "Advanced Solow model as a tool for coordination of interests of spatial economic systems' development (on the materials of the South Russian macro-region)" by Anopchenko T, Gorbaneva O, Lazareva E, Murzin A, Ougolnitsky G, 2019, Advances in Systems Science and Applications, 19(4), p. 1-13 (https://doi.org/10.25728/assa.2019.19.4.778) [9] Copyright 2019 by ASSA.

4) The depreciation coefficient of fixed assets μ_i is obtained from statistical data in direct form for each year and is averaged (Table 4).

Table 4. Parameters μ_i of the SFD regions.

Region	Parameter	Value
Rostov region	μ_1	0.40640
Volgograd region	μ_2	0.4064
Krasnodar region	μ_3	0.511
Republic of Adygea	μ_4	0.511
Astrakhan region	μ_5	0.406
Republic of Kalmykia	μ_6	0.511
Republic of Crimea	μ_7	0.712

5) Specific emissions of pollution substances during production activities into the air $B_{K_i}^{\alpha}$ and water $B_{K_i}^{w}$ are parameters of environmental technology that are regulated by the federal state. As a result, for the regions of the SFD the following values were obtained (Table 5):

Table 5. Parameters of environmental emissions of the SFD regions.

Region	Parameter	Value
Rostov region	$B_{K_1}^{\alpha}$	0.0000834
	$B_{K_1}^{w}$	0.00005365
	$B_{L_1}^{\alpha}$	0.0232252
	$B_{L_1}^{w}$	0.0756389
Volgograd region	$B_{K_2}^{\alpha}$	0.00009585
	$B_{K_2}^{w}$	0.00006303
	$B_{L_2}^{\alpha}$	0.0687909
	$B_{L_2}^{w}$	0.0617981
Krasnodar region	$B_{K_3}^{\alpha}$	0.000042518
	$B_{K_3}^{w}$	0.00013121
	$B_{L_3}^{\alpha}$	0.02400843
	$B_{L_3}^{w}$	0.22684937
Republic of Adygea	$B_{K_4}^{\alpha}$	0.0000235727
	$B_{K_4}^{w}$	0.0000818161
	$B_{L_4}^{\alpha}$	0.028614278
	$B_{L_4}^{w}$	0.10440849
Astrakhan region	$B_{K_5}^{\alpha}$	0.0000563
	$B_{K_5}^{w}$	0.0000227
	$B_{L_5}^{\alpha}$	0.185544
	$B_{L_5}^{w}$	0.085229
Republic of Kalmykia	$B_{K_6}^{\alpha}$	0.000002222
	$B_{K_6}^{w}$	0.0001432
	$B_{L_6}^{\alpha}$	0.0220744
	$B_{L_6}^{w}$	0.0896685
Republic of Crimea	$B_{K_7}^{\alpha}$	0.00000364224
	$B_{K_7}^{w}$	0.000000941469
	$B_{L_7}^{\alpha}$	0.02335533
	$B_{L_7}^{w}$	0.006049158

6) For calculation of the coefficients of efficiency of environmental assignments c_i^{α} and c_i^{w}, accordingly, with the index a for the air, with

the index w for water, data on the share of pollutant neutralization are used.

As a result, for the regions of the SFD (Table 6):

Table 6. Ecological parameters c_i^{α} and c_i^{w} of the SFD regions.

Region	Parameter	Value
Rostov region	c_1^{α}	0.000393
	c_1^{w}	0.000104
Volgograd region	c_2^{α}	0.000497
	c_2^{w}	0.000528
Krasnodar region	c_3^{α}	0.000635
	c_3^{w}	0.0000942
Republic of Adygea	c_4^{α}	0.016904
	c_4^{w}	0.002975
Astrakhan region	c_5^{α}	0.00085
	c_5^{w}	0.001107
Republic of Kalmykia	c_6^{α}	0.029967
	c_6^{w}	0.002841
Republic of Crimea	c_7^{α}	0.000772
	c_7^{w}	0.001659

7) Indices of influence of innovative activity in the regions of the Russian Federation for years 2005, 2010, 2015 were obtained as a result of previous studies based on official data from Federal State Statistics Service [49]. The chosen frequency is determined by the structure of the model and periods of stability of the economic system. Based on them, the function of the influence of innovation on the release of the final product has the following form:

$$\mathrm{A}(t) = 0.00268 \cdot t^2 - 10.774 \cdot t + 10835.328 \quad (77)$$

8) The initial values of model parameters $K_i(0)$, $L_i(0)$ and $E_i(0)$ are determined according to official data from the Federal State Statistics Service [49] on the volume of fixed assets and the size of the working population, and also by finding the ratio of Gross National Product (GNP) production to labor resources, respectively, for each region for year 2022 (Table 7).

Table 7. Initial values $K_i(0)$, $L_i(0)$ and $E_i(0)$ of the SFD regions.

Region	Parameter	Value
Rostov region	$K_1(0)$	2,786,870
	$L_1(0)$	1958.1
	$E_1(0)$	687.98
Volgograd region	$K_2(0)$	2,180,917
	$L_2(0)$	1124.6
	$E_2(0)$	685.97
Krasnodar region	$K_3(0)$	5,937,791
	$L_3(0)$	2599.1
	$E_3(0)$	856.42
Republic of Adygea	$K_4(0)$	202,111
	$L_4(0)$	152.1
	$E_4(0)$	653.56
Astrakhan region	$K_5(0)$	1,498,692
	$L_5(0)$	487.6
	$E_5(0)$	863.33
Republic of Kalmykia	$K_6(0)$	203,657
	$L_6(0)$	111.1
	$E_6(0)$	598.66
Republic of Crimea	$K_7(0)$	2,212,391
	$L_7(0)$	840.4
	$E_7(0)$	427.31

9) We accept the value of discount factor ρ based on the averaged over the period key rate (refinancing rate) of the Central Bank of the Russian Federation: $\rho = 0.094$ (Table 8).

Table 8. Discount factor values.

Years	Meaning, %
2005	13.0
2010	8.0
2015	8.25
2016	10.0
2017	7.75

Regional authorities in Russia recommend to evaluate the effectiveness of investment projects on the basis of the budget norm of discount, which is determined at the level of the average for the corresponding period refinancing rate of the Bank of Russia. As for the lower level (enterprises), the largest enterprises in terms of revenue in the regions of the SFD were selected for the study. Empirical data of the effectiveness of business entities (accounting statements) were taken from open sources (according to the RosBusinessConsulting agency) for the studying period 2015–2017. The production capacity of each enterprise was calculated from the equation $g \cdot {r_t}^{\alpha} = r_{t+1}$ where r_t is a revenue in year t, and α is the elasticity of product output depending on the type of activity (Tables 9–15).

Table 9. Economic parameters of firms of the Republic of Adygea.

Title	Industry	MFRN (Main Federal Registration Number)	Revenue, rub	Production capacity
LLC “Dortransservice”	Freight transportation	1050100637190	7.6 bn	2163.086321
EXPRESS-KUBAN LLC	Fruit and vegetable production	1020100824985	7.3 bn	1.573049862
LLC “YUG-AUTO EXPERT”	Retail trade of passenger cars	1150107011174	6.5 bn	4883.697502

Table 10. Economic parameters of firms of the Astrakhan region.

Title	Industry	MFRN	Revenue, rub	Production capacity
LLC “LUKOIL-Nizhnevolzhskneft”	Oil production	1023403432766	334.2 bn	36,132.377932
LLC “Gazprom Dobycha Astrakhan”	Gas production	1023001538460	126.5 bn	20,533.596854
PJSC “ASTRAKHAN ENERGY SALES COMPANY”	Electric power industry	1053000000041	12.4 bn	2100.167620
LLC “GAZPROM MEZHREGIONGAZ ASTRAKHAN”	Wholesale in fuel	1163025054499	9.7 bn	1642.058660

Table 11. Economic parameters of firms of the Volgograd region.

Title	Industry	MFRN	Revenue, rub	Production capacity
JSC “VTZ”	Rolled metal products	1023401997101	75.6 bn	9763.019409
LLC “LUKOIL-Volgogradneftepererabotka”	Petroleum products	1023404362662	69.8 bn	9114.898893
LLC “Gazprom Mezhregiongaz Volgograd”	Gasification	1023403844441	38.1 bn	989.4280473

Table 12. Economic parameters of firms of the Republic of Kalmykia.

Title	Industry	MFRN	Revenue, rub	Production capacity
LLC “LUKOIL-NIZHNEVOLZHSKNEFT-KALMYKIA”	Pipeline transport activities	1143015002569	4.3 bn	597.9983773
MAI“KOMMUNALNIK”	Local government bodies for managing general issues	1210800000432	4.1 bn	1.232534275
LLC “LOTOS”	Retail trade beverages	1090803000012	2.9 bn	1305.265350
LLC “888”	Retail trade food products	1080816005380	2.1 bn	998.1281008

Table 13. Economic parameters of firms of the Krasnodar region.

Title	Industry	MFRN	Revenue, rub	Production capacity
JSC “TANDER”	Retail trade food products	1022301598549	2.1 trln	472,730
LLC “ONIX COFFEE”	Restaurants and food delivery services	1192375025820	247.3 bn	84,941.085308
LLC “LUKOIL-YUGNEFTEPRODUKT”	Retail sale of motor fuel in specialized stores	1022301424254	198.6 bn	23,872.781806
LLC «Slavyansk ECO”	Petroleum products	1112370000753	161.9 bn	20,152.638582

Table 14. Economic parameters of firms of the Rostov region.

Title	Industry	MFRN	Revenue, rub	Production capacity
LLC “AKSAYVTORMET”	Utilization	1196196005191	1.6 trln	52,231.211323
LLC “RODNYE POLYA”	Wholesale trade of agricultural products	1106165002350	299.5 bn	69,985.622305
JSC “NZNP”	Petroleum products	1046151001071	175.0 bn	21,435.536144
JSC “ASTON”	Production of oils and fats	1096194001683	155.1 bn	37,052.936101

Table 15. Economic parameters of firms of the Republic of Crimea.

Title	Industry	MFRN	Revenue, rub	Production capacity
GUP of the Republic of Crimea “KRYMENERGO”	Electric power industry	1149102003423	28.0 bn	3908.531602
LLC “KEDR”	Fuel wholesale trade	1149102012905	26.5 bn	3990.784423
LLC “PUD”	Retail trade food products	1159102103093	20.5 bn	5233.130916
LLC “PARTNER”	Wholesale trade of tobacco products	1142315002830	17.7 bn	4796.881385

The principal’s strategies for each region include:

1) The share $s_i(t)$ from Gross Regional Product (GRP) that goes to production purposes;
2) Shares $v_i^a(t), v_i^w(t)$ from production investments that go towards eliminating the consequences of air and water pollution;
3) Shares $k_{ij}(t)$ from production investments that go to the general development of the macroregion and to their own development (if $i = j$). The region also allocates a share $k_{i0}(t)$ for purposes external to the SFD (federal and inter-district programs and projects).
4) The share $B_i(t)$ from GRP that goes to the assignments on PPP projects.

Leading Russian investors, represented by First Vice President of Gazprombank Alexey Chichkan, believe that infrastructure projects should be financed on the level of 4.5%–5% of GDP, while the current average financing in Russia is about 1.8%. GRP in the SFD, according to official statistics, in 2017 amounted to 5.8 trillion rubles. In this context, we consider it is necessary to set the target level of allocations for PPP projects in the SFD at a level of at least 0.261 trillion rubles.

Let’s start the study in simulation mode with the case when SFD doesn’t allocates funds for the development of PPP, in other words $B_i(t) = 0$. Let us set the conditions for sustainable development as an increase of the output of regional products for each region by 1% annually. This can be achieved with the following shares of GRP that go to production purposes (Table 16).

Table 16. Optimal production investments of the SFD regions to increase GRP by 1%.

Region	Share $s_i(t)$, %
Rostov region	76.4
Volgograd region	44.4
Krasnodar region	24.6
Republic of Adygea	22.11
Astrakhan region	79.2
Republic of Kalmykia	68.7
Republic of Crimea	28.3

In the same time, some regions of the SFD can provide resource assistance to neighboring regions that need in cooperation for solution of their tasks, in the following amounts from their production investments (Table 17).

Table 17. Optimal interaction coefficients of the SFD regions to increase GRP by 1%.

Donor	Recipient	Share of $s_i(t)$, %
Rostov region	Volgograd region	30
	Krasnodar region	10
	Astrakhan region	30
	Republic of Crimea	10
Republic of Adygea	Astrakhan region	10
	Republic of Kalmykia	10
	Republic of Crimea	30
Republic of Kalmykia	Rostov region	30
	Republic of Adygea	20
	Republic of Crimea	10

Notice that such economically weak regions as Adygea and Kalmykia can afford themselves assistance to other regions. Their GRP level is relatively low, which requires less effort to increase this indicator by 1% by directing available funds to projects in other regions.

In this case, the gains of the regions are equal to zero, because at each stage they are proportional to the production of a public good during the development of PPP, which does not exist in this case. The condition of sustainable development for realization of a public good as a result of the realization of PPP projects is not satisfied. But in the same time, funds remain for consumption and the total gain of the macroregion is equal to 64.3281 million rubles.

To increase the output of regional products by 2%, it is necessary to allocate for production purposes the following parts of the GRP (Table 18).

Table 18. Optimal production investments of the SFD regions to increase GRP by 2%.

Region	Share $s_i(t)$, %
Rostov region	86
Volgograd region	93
Krasnodar region	85
Republic of Adygea	22.4
Astrakhan region	89
Republic of Kalmykia	56.6
Rostov region	0

In the same time, some regions of the SFD can provide a resource assistance to neighboring regions that need in cooperation for solution of their tasks in the following amounts from their production investments (Table 19).

Table 19. Optimal interaction coefficients of the SFD regions to increase GRP by 2%.

Donor	Recipient	Share from $s_i(t)$, %
Rostov region	Volgograd region	30
	Astrakhan region	30
Krasnodar region	Rostov region	20
	Astrakhan region	20

In this case, economically weak regions cannot help the rest, since increasing their GRP by 2% requires more efforts. The main burden in this case is forced to bear by the two most economically developed regions of the SFD: the Rostov region and the Krasnodar region.

To increase the GRP of all regions of the SFD by 3% is not possible. In the case of directing 10% of the GRP of each region to the development of PPP, the situation has the following dynamics: the share of the GRP allocated for production purposes is equal to zero, and not a single region needs the help of another region, relying only on business funds. In the same time, the GRP of all regions can be increased by 50% in five years. Consumption of the macroregion will decrease in comparison to the case of complete absence of support for PPP projects and will amount 63.49 million rubles. In this case, the total output of the public good is equal to 765.23 billion rubles. But the condition for sustainable development for the production of public good as a result of the realization of PPP projects will also not be satisfied.

In the same time, if 10% of all resources are allocated to the development of PPP, the GRP of the regions will be able to increase by 60% in five years. The share of GRP allocated for production purposes will be equal to zero, and not a single region will require assistance by attracting business funds. Consumption of the macroregion will decrease even further and amount to 56.43 million rubles. In this case, the total output of the public good will be 1.53 trillion rubles. The condition of sustainable development concerning the production of public goods in the result of realization of PPP projects is performed.

The practical expression of such cooperation can be realized in the initiation of PPP projects for the development of transport and logistics infrastructure in border areas for cargo-passenger transportation in the direction from Western Europe to Central Asia. Thus, among the perspective projects, can be taken to the realization projects of road routes to the direction of the Povolzhye, the Ural and Siberia, and also countries of Commonwealth of Independent States: TSA 4 Saratov–Volgograd–Rostov-on-Don–Novorossiysk; NSA 6 Lugansk–Kamensk-Shakhtinsky, Donetsk–Matveev Kurgan–Rostov-on-Don; reconstruction of routes P 61-1 Rostov-on-Don–Stavropol and further in the direction of Budennovsk–Kochubey–Makhachkala with exits to Elista, Astrakhan, and Kazakhstan.

In the environmental projects, it is possible to implement joint interregional efforts on utilization of production and consumption waste, liquidation of accumulated environmental damage, water pollution, conservation of biological diversity, protection and reproduction forests, counteraction soil degradation, monitoring the condition of air basins. Border PPP objects may act on the territories of restructuring of the coal industry in the Eastern Donbass, areas of the Volgodonsk industrial zone, territories of burial of toxic chemicals and pesticides of the associations “Selkhoztekhnika” (since 1975) of the Krasnodar and Stavropol territories, the Republic of Kalmykia, the Rostov region, the territories of the water protection zone of large rivers and tributaries of the Volga and Don, coastal territories of the Taganrog Bay, the Azov Sea, specially protected natural areas of the state natural biosphere reserve “Rostovsky”, the Federal natural reserve “Tsimlyansky” and others.

DISCUSSION

The proposed model develops the model previously discussed in [9,10]. The model in [9,10] is a special case in which the PPP sector is not considered. The results of investigation of the current model in the absence of investments in PPP (i.e., with $B_i(t) = 0$) completely coincide with the results of the study of the model in [9,10].

Namely, in the absence of investments in PPP projects, the results of modeling are the following. All regions will simultaneously be able to increase GRP by 2% and reduce pollution by 7% in comparison with the original values. The Volgograd region and the Astrakhan region should send almost all funds to replenishment of the fixed assets, leaving a small part for consumption. At the same time, the Astrakhan region does not have enough of its own funds for increase the GRP, and the Rostov region should help it by directing 40% of its investments to the development of the neighboring region. The least amount of funds for investment in development can allocate Republic of Adygea: 22%. All other regions invest 55%–60%.

It is not possible to increase the GRP of all regions of the SFD by 3% in comparison with the last table because the Rostov region will have to allocate all funds to help the Astrakhan region. There may be a situation for a simultaneous increase of the GRP by 3% for all regions except the Astrakhan region, while it will be able to increase GRP only by 1%. In the same time, the Volgograd region, the Republic of Crimea and the Republic of Kalmykia must allocate to investments 87% of budget resources. The least of all for investments can allocate the Republic of Adygea—35%, other regions—about 80%. The Rostov region should send to the Volgograd region 40% of its investments.

A maximization of the GRP is possible by 8% for the Republic of Adygea, by 7% for the Rostov region, by 4% for the Republic of Kalmykia and the Republic of Crimea, by 3% for the Krasnodar region, by 2% for the Volgograd region, by 1% for the Astrakhan region. In this case, the Krasnodar region must allocate for investment in the Rostov region 20% of its resources. It is noteworthy that in this case the Krasnodar region will not have enough funds to increase the GRP from 3% to 4%, but there are enough resources to increase the GRP of the Rostov region from 5% to 7%.

A further increase of the GRP in the absence of investments in the development of PPP projects is impossible. In allocating 10% of the funds from GRP received at the previous stage of the development of PPP projects, GRP during five years can be increased by 50%, and by allocating 20% of funds for the development of PPP projects, GRP can be increased by 60% in comparison with the available value at the initial moment of time. In the same time, the regions gains independence from other regions of the Southern Federal District and can cope with its socio-economic development without their help.

There are some limitations of the research, namely:

a. The study assumes homogeneity within regions, which can be problematic as it overlooks intra-regional variations in economic activities, environmental policies, and industrial practices.
b. The model appears to be focused on short-term equilibrium states and does not adequately address long-term sustainability or the potential for structural changes in the economy or environment.
c. The study uses averaged and historical data for various parameters, which may not capture recent changes or trends, especially in fast-evolving areas like environmental technology.

We plan to get rid of these limitations in our future research.

As for validation, we base our analysis on the Solow model. Its verification is described in [50]. It is much harder to validate a modified game theoretic model because it requires to apply the described strategies in regional development. We think that the main criterion of truth is practice. If the experts in a problem domain will agree that the model results are adequate then there they are.

The priority of the development formats of PPP in interregional cooperation in the South of Russia is due to insufficient support of these forms of implementation of significant projects from government structures, great business potential, stable investment attractiveness, favorable geographical location due to the intersection of several transport corridors, many areas of mutually beneficial activities in the field of environmental protection, water infrastructure, transport connectivity of territories, social and economic directions.

Examples of effective realization of PPP projects are already known in Russia.

The government of Moscow City and the Moscow region are realizing large-scale projects of road concessions: “Northern understudy of Kutuzovsky Prospekt” (Bagration Avenue), reconstruction of transport interchange hubs “Butovo–Zheleznodorozhny” (Southern understudy of the Moscow Railway Road), “Moscow–Kasimov” (understudy of Yegoryevskoye Highway) and “Vinogradovo–Tarasovka” (connection of Dmitrovskoye and Yaroslavskoye highways), integrated development of territories. In Saint Petersburg and the Leningrad Region a concession agreement for the construction of a latitudinal expressway, the “Western High-Speed Diameter”, Pulkovo airport, projects for the development of a tram network, reconstruction of a perinatal center and a multifunctional sports and leisure complex are implemented. In the Nizhegorodskoy region several concession agreements are implemented, among which the largest is in relation to the understudy of Gagarin Avenue in Nizhny Novgorod. In the Volgograd region a project of creation sanatorium and resort complex “Elton” in the Pallasovsky district is realized.

The Rostov region and the Krasnodar region occupy 4th and 5th places in Russia by volume of private investments. Among the perspective areas for development of PPP, should be highlighted the project of revival of high-speed river passenger transportation based on the vessels “Valdai” and “Meteor” along the river Don and the Azov Sea. An important area of PPP projects in South of Russia is the touristic industry, included the development of resorts in the Krasnodar region and the Republic of Crimea in the water areas of the Azov and Black Seas.

In the projects of high social significance, an accounting and effective coordination of interests all active agents is extremely important. As a tool for solving this problem, it is proposed to use the mechanism of an expanded tripartite contract between government authorities, the business community and public organizations. This mechanism is fundamentally different from the traditional PPP scheme by including in it observers from public organizations, which makes it possible to increase the social and environmental responsibility of the sides in realization of projects and as fully as possible take into account the interests of society during their realization (Figure 2).

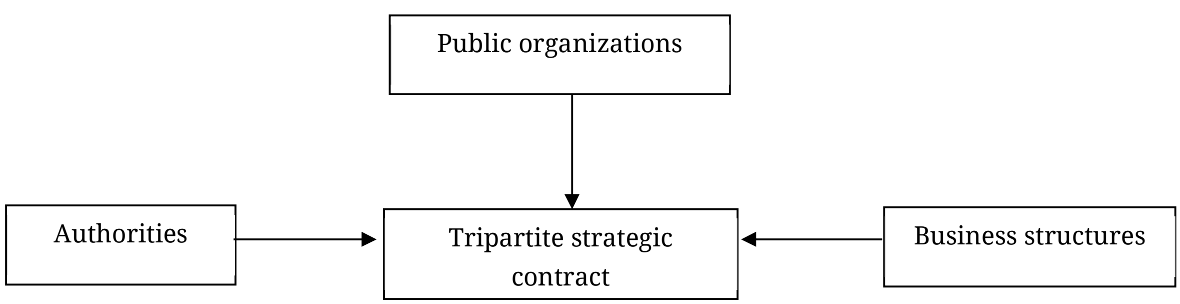

Figure 2. Scheme of a strategic contract in the realization projects of extended partnership.

This forms a fundamentally new instrument of expanded public-state-private partnership, focused on realization of structural transformations of urban environment on the base of a tripartite strategic contract.

At present, there are ideological obstacles to increasing the volume of partnership between government and business for solution of the important socio-economic tasks. State structures should realize all advantages and effects of attracting private investments, and entrepreneurs should learn to trust the authority, receive guarantees of sustainable relationships for the perspective and long-term contracts. The presented results of research demonstrate an effectiveness of interregional cooperation and make it possible qualitatively harmonize the interests of all participants in the cooperation.

As general recommendations we can note:

1) a unification of business communities for representation their interests and strengthening political influence in the process of interaction with the authority;
2) a formation of unbiased interest from government agencies to partnership with business, shaping of special tax regimes and other benefits for attraction of investors;
3) an active participation of civil society in monitoring the quality of public goods generated during the implementation of PPP projects.

Summarizing the experience of Great Britain, France and Canada—the leading countries in the development of PPPs, taking into account significant national characteristics and starting conditions, we can identify several limiting factors for Russia and outline ways of overcoming them:

1) legal environment—development of normative right-wing base for regulating relations, delimiting powers, taking into account the interests and securing guarantees for the participants of partnership on all levels of management;
2) development of institutions—the formation of the bodies of coordination on the level of the federal center and regional entities performing functions of managing the development of public-private partnerships, endowed with the appropriate power of distributing resources of support;
3) business traditions—determining the conditions for long-term trustful partnership, mutual support of business and government, strategic reliability of the implementation of socially significant projects, planning directions of cooperation;
4) economic development—stabilization of the macroeconomic environment, increasing the level of investment attractiveness of regional projects, objective analysis of projects on economic indicators of added value, level of risk, costs and financial flows;
5) political factors—transformation of public policy in the direction of "new public management", creation of conditions for transparency and openness for society and business taken by decisions, the possibility of influencing to the choice of implementation infrastructure projects.

It should be mentioned that the aspects of success of PPP projects in the world widely studied, however, for Russia these issues are still valid due to the insignificant volume of national specifics of models for the implementation of such projects, the small volume of implemented PPP projects in the structure of GDP (GRP) and not so successful their implementation compared to leading countries.

The justified application of the proposed forms of PPP will increase overall efficiency of this model of cooperation between society, the state and business, which should have a beneficial effect on the level of development of the partner infrastructure, and therefore increase competitiveness.

CONCLUSION

We have investigated a dynamic Stackelberg game theoretic model of a regional social-economic development with consideration of interests of the stakeholders. We studies four possible combinations of linear and power concave model production functions. Two cases allow for a complete analytical investigation while two other cases do not. It is possible to solve the problem described by equations (29), (30) given $s_{ij}(t)$ or the problem described by equations (31), (8) given $r_{ij}(t)$. From an economic point of view that means a separate use of mechanisms of stimulation by resource allocation or by participation in the common income. We leave this idea for our future investigations.

In general, we used simulation modeling for numerical analysis. The results are received for the Southern Federal District of the Russian Federation. For identification of the model parameters we used official statistical data. Some recommendations on regional management are formulated. We have given a comparative characteristics of the received results with the previous results from [9,10] where we did not consider public-private partnership mechanisms.

Presented results of modeling clearly show the positive influence of PPP tools on the dynamics of macro-regional development. The proposed concept of a strategic contract in the implementation projects of extended partnership with the controlling function of the consumers of public goods can act as the subject of future research and modeling.

Russian authorities and entrepreneurs are in incomparably worse conditions of the access to resources of development due to the most severe sanctions in comparison to other developed countries, but despite these circumstances they achieve economic efficiency and shape conditions for the growth of the national economy, largely thanks to joint efforts. This potential is far from being exhausted but for the further development requires coordinated, concerted efforts of all agents of the macro-regional system.

DATA AVAILABILITY

The dataset of the study is available from the authors upon reasonable request.

AUTHORS' CONTRIBUTIONS

OIG performed the experiments and made the simulations. ADM analyzed the data. GAO designed the study. The paper was written with input from all authors.

CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

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